Ratios, Proportion and Rates

A **ratio** compares values. It says how much of one thing there is compared to another thing.

3 : 1

3 apples to 1 pear

Ratios can be shown in different ways:

Using a colon: 3 : 1

Using the word "to": 3 to 1

As a fraction: $\frac{3}{1}$

You can **scale** ratios so that they are bigger or smaller.

Example: 3 : 1 = 6 : 2 = 15 : 5

When scaling ratios, you always multiply or divide **by the same number.**

Example: 4 : 5 is the same as 4x2 : 5x2 = 8 : 10

Ratios can be **part-to-part** or **part-to-whole**.

Part-to-part ratios compare one part to another part (3 apples to 1 pear).

Part-to-whole ratios compare a part to a whole.

Example: 3 apples to 4 fruits

Example 1: If you have 6 apples, 5 bananas and 4 oranges, what is the ratio of apples to oranges? Oranges to bananas? Bananas to all the fruit? Apples to bananas to oranges?

Since the question asks "apples *to* oranges" we know the apple value will be first and the orange value will be second.

6 apples : 4 oranges

Now, we simplify by dividing both sides by their common factor of 2.

(6 ÷ 2) : (4 ÷ 2) = 3 apples : 2 oranges

For oranges to bananas, oranges will be the first number & bananas will be the second.

4 oranges : 5 bananas

Since 4 and 5 don't have a common factor, this is our ratio.

For bananas to all fruit, bananas will be the first number, all fruit the second.

First, we find the total number of fruit: 6 apples + 5 bananas + 4 oranges = 15 fruit

Now, the bananas to fruit ratio is 5 bananas : 15 fruit.

Now divide both sides by the common factor which is 5.

1 banana : 3 fruit

Now we're calculating the ratio of apples to bananas to oranges, so apples will be first, bananas second and oranges third.

6 apples : 5 bananas : 4 oranges

These three numbers don't have a common factor, so the final ratio is 6 : 5 : 4

A **proportion** is when two ratios equal each other. To things are said to be "in proportion" or "proportional" when their ratios are equal.

Example: 1 : 3 = 2 : 6

These ratios are equal, so they are in proportion.

When calculating proportions there are 2 methods you can use:

1. **Cross Multiply and Divide**

$$\frac{x}{4} = \frac{9}{6}$$

First, cross-multiply the numbers in the known corners. In this case, 4 and 9. 4 x 9 = 36

Then divide by the 3rd number, which in this case is 6. 36 ÷ 6 = 6.

2. **Think about what it would be for 1 or some other easy unit, and then multiply to get what is asked.**

If a car goes 240 km in 5 hours, how far will it go in 4 hours?

First figure out how far the car goes in one hour.

240 km ÷ 5 hours = 48 km per hour

Now multiply 48 by 4 to find out how far the car goes in 4 hours.

48 x 4 = 192 km in 4 hours.

You can use proportions to solve percents.

Example: 25% = 25/100 which is a ratio!

To solve percents using proportions:

First, put what you know into this form

$$\frac{Part}{Whole} = \frac{Percent}{100}$$

Example 2: What is 25% of 160?

The percent is 25, the whole is 160, and we want to find the part.

$$\frac{Part}{160} = \frac{25}{100}$$

Multiply across the known corners, then divide by the 3rd number.

Part = (160 x 25) ÷ 100 = 4000 ÷ 100 = 40

Answer: 25% of 160 is 40

You could also solve by calculating it this way:

Part = 160 x (25 ÷ 100) = 160 x 0.25 = 40

Example 3: A recipe for blueberry muffins calls for 2 cups of flour, 2 eggs and 1 cup sugar. If you have 5 cups of flour, how much sugar and how many eggs should you use?

First, write the out the ratios from the original recipe:

2 cups flour : 2 eggs : 1 cup sugar

Now, write the ratios of the form:

$$\frac{original}{new} : \frac{original}{new} : \frac{original}{new}$$

$$\frac{2}{5} : \frac{2}{x} : \frac{1}{y}$$

Now, what do you have to multiply 2 by to get 5? We can answer this by dividing 5 by 2.

5 ÷ 2 = 2.5

So now we multiply everything by 2.5

(2 x 2.5) cups flour: (2 x 2.5) eggs : (1 x 2.5) cup sugar

= 5 cups flour : 5 eggs : 2.5 cups sugar

Answer: You would use 5 eggs and 2.5 cups sugar.

A **rate** is a ratio where the two quantities have different units.

Examples: 40 miles per hour, 3 girls to 4 boys, 50 words per minute

Real World Applications of Ratios, Proportions, and Rate

- speed: km/hour, miles/hour
- Cooking: doubling, or tripling recipes
- Gas prices